State-space curvatures and self-organizing criticalities

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Abstract

Self-organizing criticalities are a much studied notion, within disciplines ranging from ecosystems/living systems to economic systems and markets. But there is still no consensus or general framework for explaining the 'spontaneous' emergence of this kind of 'orderly' behavior.
This paper generalizes the second law of thermodynamics to dynamic state-spaces with increasing dimensionality and introduces the notion of *spate-space curvature*, in order to provide such a framework.

1 Introduction

The reader is expected to be familiar with the fact that the second law of thermodynamics is often popularized as stating that (a certain class of) systems tend to move from order to disorder (showing an increase in entropy), and that the most notable alleged exception to this is the spontaneous emergence of life-forms (so-called Self-organizing Criticalities (SoC))[1] [2]: somehow life seems to defy this law, at least for a while. This has puzzled scientists in a wide array of sciences for decades, with Schrödingers 'What is Life' being perhaps the most notable.
Neuroscientist Karl Friston provided a descriptive framework with his Free Energy Principle, stating that living entities, cells, brains, etc (defined by Markov blankets) tend towards a state of minimal free energy, by mirroring its environment.[3]
This framework has proven to be extremely successful in describing many biological processes and has been applied to a wide range of disciplines, including machine learning. However, this framework is only descriptive and it does not explain *why* it minimizes its free energy. As such, a general explanatory or even predictive theory is still being sought after.

Difficulties in thinking about entropy

One of the reasons that a general explanatory theory has not been found yet is that it is very hard to properly think about what is going on. Intuitive notions like 'order' and 'disorder' or 'chaos' are extremely contextual and subjective concepts. As such the order/disorder-dichotomy is useless for formally classifying dynamic behavior in systems. This human subjective bias in looking at systems is so deep that it is very hard to recognize flaws in ones thinking, and in failing to do so some even state that the second law of thermodynamics does not actually hold universally, and should be challenged. Maxwell's demon is the most famous example of supposedly challenging the Second Law. But this paper argues that it is never the case of a flaw in the Second Law, it is always a flaw in one's thinking about it.
Most paradoxes come from thinking-errors that concern not properly recognizing 'the closed system' (Maxwell conveniently forgot that his demon, in monitoring the gas-molecules, is itself actually burning a lot of energy while breathing, sweating, looking, processing, and working the box, let alone its supporting metabolism and even its initial growth and learning to be able to work the box to begin with, which, including all this in the system, easily increases aggregate entropy after all, in full conformance to the Second Law. - As if a demon would do it for free..).

2 Materials and Methods

2.1 Systems, micro/macro-states

We need to think about dynamic systems at a higher level of abstraction, in order to be able to generalize what we are talking about. This is done best by the use of the mathematical notions of 'microstate' and 'macrostate' of a system. The Second Law, stating that entropy will increase over time (or at least remain constant), is actually a statistical inference: over time it is more likely that a system will occupy a macrostate which can be represented by many microstates, than a macrostate with little microstates. This inference actually not only tells you that entropy will increase over time, it also tells you that it will increase *as fast as possible* (the Maximum Entropy Production Principle) [5] [4].
The laws of thermodynamics were formulated for physical, ideal gases. If we want to *generalize* the laws to other systems, it is imperative to be strictly precise in what we choose and define as 'the system', and 'the macrostate and microstate', and *then* check if we can define a meaningful entropic gradient over time (for which we can statistically infer that it can never be negative).

2.2 State-spaces

That can be challenging for many domains, but even more so when it comes to 'living systems'. In the myriad of currently defined types of entropies, ranging from economics to information theory and the physical sciences, the associated state-space is always treated as a given, a constant, an unchanging environment; the state-space is just the state-space. It is a fixed host.
In order to explain that the 'spontaneous emergence' does actually NOT defy the second law, we first need a generalization of this 'fixed state-space' as a special case of 'a changing state-space'.
Mathematically this translates to a *dynamic dimensionality of the state-space*, instead of the canonical fixed dimensionality.
In expressing entropy mathematically, that would result in just adding another parameter or function that accounts for the dimensionality of the system at time t, but for a better understanding we will keep using the notion of 'a state-space' that has its own dynamic, which in turn hosts the state of 'the system' at hand.

2.3 State-space curvature

In a state-space of constant dimensionality it would indeed be impossible for life-forms to emerge and remain stable for some time.
However, if you allow for the state-space (of the life-form) to *increase in dimensionality*, this provides for an alternative entropic gradient.
And if this dimensionality increases fast enough, this gradient may *outperform* the 'canonical' entropic gradient (on which the organism would fall apart). After all, entropy will not only increase, but it will increase *as fast as possible.* The system prefers the steepest local gradient available from its current state.

2.4 State-space curvature in real-world systems

Such an increasing state-space dimensionality actually occurs by the continuous increase of complexity of a growing organism (new cells, the whole developmental biology process actually, etc), or, for example, the increasing (economic) complexity of a growing economy.
Mathematically, since we are dealing with nonlinear complex-dynamic systems, the increase of complexity can yield chaotic attractors of entropy production. This allows us to introduce the notion of ***state-space curvature***: this increase of complexity can, in ideal cases, provide a curvature-dynamic that 'catches' the system, just like Einsteins dynamic 'spacetime curvature' can catch a body of matter.

> *The analogy sits in that for us observers the earth seems to attract the moon, but we are actually witnessing spacetime curva-*

ture; and just like that for us observers a living system seems to decrease (or resist its increase) in entropy, but we are actually witnessing state-space curvature.

So a living organism (any SoC actually) does not actually self-organize or self-sustain, but it is being organized/sustained by its state-space curvature. In other words, the 'hosting system' provides an increasing entropic gradient.

2.5 The organism is not the system

Although we now have a more generalized view on SoCs which provides a full explanation of its emergence (we generalized the canonical fixed-dimensional state-space as a special case of dynamic state-spaces), we still have an issue. After all: in order for complexity to keep increasing (to maintain the entropic gradient), the organism would need to keep growing. And it is obvious that although most living systems do grow for some time, at some point they actually stop growing, and the entropy of the organism does not increase anymore. So why does the organism not collapse as soon as it stops growing? After all, the statespace-curvature has flattened.
In order to understand this, we have to distinguish between 'the system' (caught in the state-space curvature) and 'the organism' in a strict way. Mathematically we can define 'the system' as a constant concept, but 'the living organism' is not really a fixed/constant entity at all: it breathes, sweats, eats, drinks, etc. Every other time-instance the actual material composition of 'the organism' will be different: molecules are constantly being added and substracted from the organism. As such, *the organism is not the system.* For us humans the difference is imperceivable, but this distinction is imperative to make in order to fully understand what is going on.
The dynamics of the biological composition of 'an organism' has a strong analogy with a wave at sea: the top of the wave is analogous to 'the organism', and this top constantly changes in material composition. If you define 'the system' to be the organism (the top of a wave) at time $T = 0$, then at $T = 1$ 'the system' will only have some of its particles still at the top of the wave, and the rest is flushing away in the sea, in full conformance to the Second Law. 'The organism' is NOT 'the system'.
We can now understand that the life-span of an organism can be understood as a wave on some state-space curvature, and this analogy even continues until the death of the organism: mathematically it is the same dynamic as a breaking wave.

2.6 Fractality of systems

It will be evident that, if we think of a state-space curvature, the state-space itself probably classifies as a complex-dynamic system as well. Choosing what we define as 'a system' (an organism, a cell, an ecosystem, a tornado,

a weather-system, an economic market, etc) is always arbitrary. After all, the dynamics of the system and the dynamics of state-space influence each other, so you can also think of them as a single dynamic system.
You can define 'a tornado' as a system and then identify the hosting weather-system as a dynamic state-space. In this case it is clear that determination of which molecules would be part of the tornado-system is futile: it would change every microsecond. This is the reason that the mathematical field of nonlinear, complex dynamic systems does not deal with particles or cells or entities, but only with their dynamics. And these dynamics can show patterns like chaotic attractors, and classifiable bifurcations, etc. This actually tells us that what we think of as the conception and death of a discrete organism, are actually bifurcations within the larger, hosting complex-dynamic system (an ecosystem). From this systems-perspective ***there is no discrete organism.***, but only an complex-dynamic system with myriads of chaotic attractors of bifurcating into and out of existence, like eddies in a river.
That helps us to overcome our bias (from the human perspective and scale) on what is generally meaningful to define in order to get a better understanding.

2.7 Numerical example

To demonstrate that the entropy-gradient towards a higher dimension is greater than the gradient within the same dimension, we use a simple system existing of a grid-based D-dimensional space of size m^D, which can be occupied by n particles.
The statespace then consists of all permutations of these particles across this D-dimensional grid, so every permutation is a microstate. We define the macrostate to be the average distance between all particles p:

$$\overline{d} = \frac{1}{n^2 - n} \sum_{i=1}^{n} \sum_{j=1}^{n} d(p_i, p_j) \text{with } i \neq j$$

The microstate with absolute *minimal entropy* is the single permutation with maximum average distance (every particle in its own corner), and the most common average distance (*maximum entropy*) are the states with every particle evenly distributed in its own 'quadrant'. For a 2D-grid with 4 particles, the following micro-states can be identified (dark/light-green both represent a different example of a microstate for the given average distance):

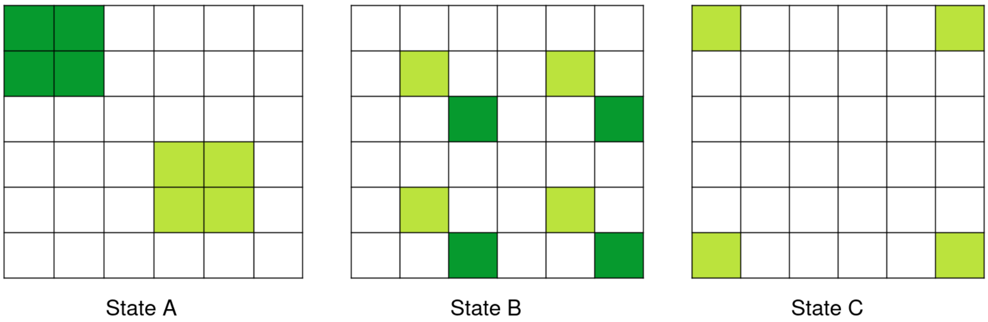

Fig 1: State A with $\overline{d}_{min}$, B with $\overline{d}_{opt}$, C with $\overline{d}_{max}$

The number of microstates Ω is given below for a 2D-grid (6×6) and a 3D-grid ($6 \times 6 \times 6$) is given below.

dimension	$\overline{d}_{min}$	$\overline{d}_{opt}$	$\overline{d}_{max}$	$\Omega\overline{d}_{min}$	$\Omega\overline{d}_{opt}$	$\Omega\overline{d}_{max}$
2	1.13	3.41	5.7	25	9^4	1
3	1.2	4.13	6.9	450	54^4	6

If we define the state-transition for this 'system' to be any permutation for which all 'populated' cells can move only to adjacing cells relative to their current position, we can infer from the numbers that for any permutation that occurs from an initial state within a 2D-plane, it is much more likely that the system will traverse into the 3D-grid, than that it will remain within the original 2D-plane. This is also intuitively trivial.

2.8 Main thesis

Traditionally, we have defined 'complex dynamic systems' as systems with a fixed statespace, with a certain dimensionality. The canonical entropic gradient is defined over time: ∇S_t. We can now generalize this notion to 'hypersystems', which can *change in dimensionality* of its statespace over time. For hypersystems that increase in state-space dimensionality D over time, we can introduce a generalized entropic gradient by expanding into the temporal-dimensional plane:

$$\nabla S_{t,D} \tag{1}$$

This allows for a comparison between the entropic gradient on the time-axis (partial derivative) and the gradient on *both* time and statespace-dimension axis. The main thesis of this paper is that the entropic gradient towards a higher dimensional statespace is always larger than the gradient at current dimensionality. This can then be formulated as:

$$\frac{\partial S}{\partial D \partial t} > \frac{\partial S}{\partial t} \tag{2}$$

Roughly speaking, for many real-world systems entropy will increase *logistically* towards some equilibrium with maximum entropy. An illustration of the principle of the main thesis would then look like the figure below.

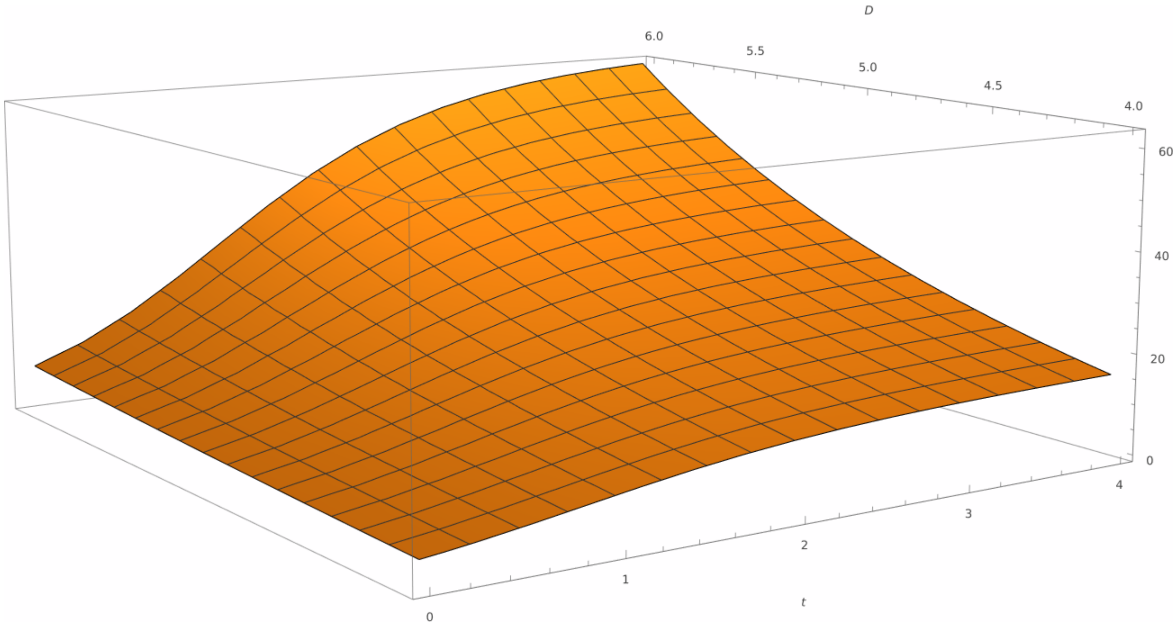

Fig 2: Temporal-dimensional entropy trajectory-plane of a hypersystem

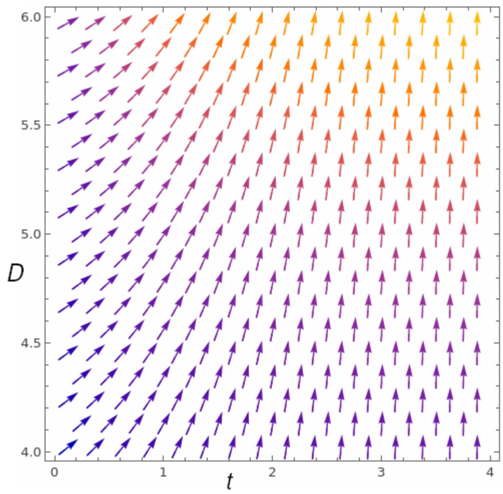

Fig 3: Vectorfield of (t against D) associated with figure 2

The figure shows that, at any point, the gradient along the t-axis will be outperformed by the gradient along both the t-axis and D-axis. Entropy can maximize faster by traversing along both the t-axis and the D-axis, and given the Maximum Entropy Production Principle, it will.

We distinguish between the hypersystem statespace-dimensionality at time t called D_t, and the dimensionality of the occupied state of the system at time t, called d_t. Given that real-world systems show some inertia in state-transition, D_t can increase faster than d_t.
Digital markets are an example: transactional complexity has increased so much and so fast, yielding a fast increase in statespace-dimensionality, that actual market-dynamics still lag behind in optimizing towards higher market-entropy production rates. For this timeperiod, one could argue that not only statespace-dimensionality far outpaces the state-dimensionality, but also the increase of statespace-dimensionality far outpaces the increase of state-dimensionality:

$$D_t > d_t \tag{3}$$

and

$$\frac{\partial D}{\partial t} > \frac{\partial d}{\partial t}\ , t \in \text{boomingperiod} \tag{4}$$

In such a booming-period the increase in entropy does *not* result in a trend that we conveniently call 'from order to chaos', but 'from chaos to order' (i.e. an increase in complexity).
So the spontaneous emergence of a Self-organizing Criticality is not actually self-organizing, but an extreme increase in statespace-dimensionality, and the system perfectly follows the second law of thermodynamics.

3 Results, applicability

This paper has provided a generalization of the thermodynamics for complex-dynamic systems. The applicability of this generalization is not limited to 'living systems'. It also applies to other domains, such as weather-systems: for example: a tornado does not self-organize, but the hosting weather-system provides the entropic gradient that provides for a chaotic attractor for nonlinear dissipation of temperature/pressure/humidity differences. The tornado emerges and disappears like a wave in the sea. Of course the dimensionality of the state-space for living organisms is orders of magnitudes higher than the dimensionality of the tornado, but the mathematical principle is exactly the same.
Another application concerns 'the economic system', ranging from transactional micro-market-structures (the quant-domain) to traditional economic growth and macro-economics. Yes the economic system has thermodynamic aspects. Not concerning entropic dissipation of money (this does not hold), but transactional entropy concerning *settlement of suppy and demand*: a market-system 'wants' to settle (dissipate) supply and demand as much and

as fast as possible. Chaotic non-linearity optimizes through *infrastructural clustering* of settlement (exchanges) and (with higher dimensionality from increasing complexity (IT-revolution)) even bifurcating towards *temporal clustering* of settlement (High Frequency Traders).
Also, recently, a lot is going on around solving the black hole paradox in relation to entropy, for which state-space curvature through increase of complexity provides a full explanation.

4 Discussion

There is a large class of complex-dynamic systems which are fundamentally driven by the maximization of entropy production. Recognizing the patterns at this high level of abstraction can provide a much better understanding of their behavior.
The generalization of the state-space of a system as a special case of dynamic state-spaces allows us to introduce the notion of *state-space curvature*. This curvature can provide for a specific *entropic gradient in dimensionality* which outperforms the 'canonical' entropic gradient (which would degrade the system towards disorder), and as such can yield and sustain an 'orderly' pattern within a hypersystem. Self-organizing Criticalities do not really self-organize, but are floating patterns on these curvatures. These patterns are what we coveniently recognize as 'living systems', at any scale, but they do not formally classify as a discrete, dissipative system. Organisms do NOT minimize free energy (autonomously), but they are patterns in a hypersystem that maximizes entropy production.
This synthesis of the statistical Principle of Maximum Entropy Production and complex-dynamic systems provides for a fully explanatory framework for the 'spontaneous' emergence and sustainability of emergent, orderly patterns in non-linear, chaotic real-world systems, ranging from living systems to economic systems.

References

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