Principle of Maximum Entropy Production and a generalization of statespaces

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June 2020

Abstract

The *maximization of entropy* S is accepted as an inevitability (as the second law of thermodynamics) by statistical inference alone. The *Maximum Entropy Production Principle* (MEPP) states that not only S maximizes, but $\dot{S}$ as well: a system will dissipate as fast as possible. There is still no consensus on the general validity of the MEPP even though it shows remarkable explanatory power (both qualitatively and quantitatively), and has been empirically demonstrated for many domains.
In this theoretical paper I provide a generalization of state-spaces, and then show that the maximization of $\dot{S}$ actually comes from the same statistical inference, as that of the 2nd law of thermodynamics.
For this generalization, I introduce the concept of the *poly-dimensional microstate-density of a statespace*. This concept also allows for the abstraction of 'Self Organizing Criticality' to a bifurcating local difference in this density.
Ultimately, the inevitability of entropy production maximization has significant implications for the models we use in developing and monitoring socio-economic and financial policies, explaining organic life at any scale, and in curbing the growth of our technological progress, to name a few areas.

Introduction

In the last few decades, several attempts have been made to explain the occurrence and stability of life-forms, or living systems. Most notably, the alleged conflict between 'autopoiesis' and the second law of thermodynamics is still unsolved [PN71]. Despite the impressive progress and multidisciplinary approaches in this field, there is still no conclusive consensus on any of these aspects [RBF17].

This theoretical paper does not present yet another new theory to explain life, but rather a generalization of our existing theoretical frameworks, that

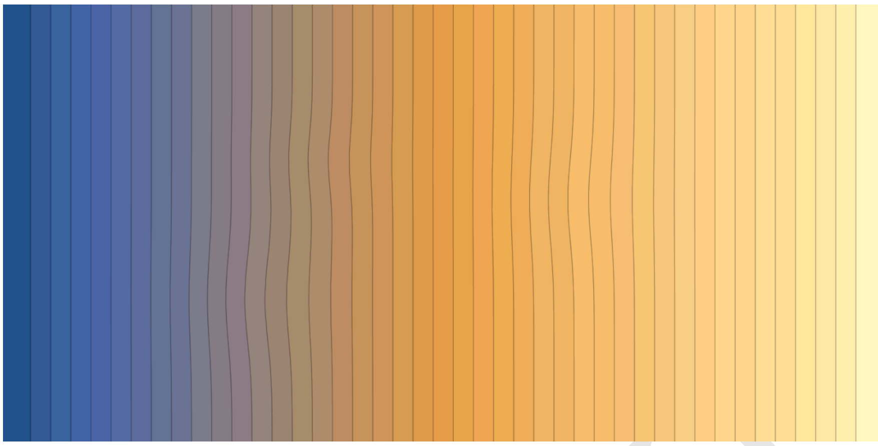

Fig 1: Contourmap of W in a monotonically increasing statespace-density

provides for a simple yet elegant demonstration of the inevitability of the occurrence of Self Organizing Criticalities, in a certain class of statespaces.

The maximization of entropy

The well-known Boltzmann equation for entropy in a closed system is:

$$S = -k_B \ln W \tag{1}$$

where W indicates the number of *microstates* that correspond to a certain *macrostate*.
The well-known Second Law of Thermodynamics tells us that the state of a system will tend towards macrostates with a higher W. The trivial example of such a macrostate is a homogeneous distribution.
It's irreversibility comes from statistical inference: the state of a system has a higher probability to have a macrostate with many microstates, than a macrostate with little microstates. Therefore, over time, it's W (and S) will increase up to some maximum (see Figure 1).

The maximization of entropy-production

Not only will a system maximize its entropy S, it will do so as fast as possible (maximization of $\dot{S}$). This is called the 'principle of maximum entropy production' [Dew03] [Dew05]. This principle has been empirically validated for many different domains like the atmosphere [Pal75] [Kle10] [OOLP03], crystallization [Hil90], enzyme reactions [DVBF17] and the economy [TMS20]. Intuitively this makes sense, but this principle is still debated, and its ubiquitous validity has not been demonstrated yet [Pal05] [Mar10] [MS14].

A generalization of state-spaces

By convention we consider the state-space of a system having some dimensionality n, so we can describe the state of a system as some $w \in \mathbb{R}^n$, for example. This dimensionality is also a measure of complexity in a non-linear dynamic system.
We now introduce the concept of a *poly-dimensional state-space*, i.e. a single state-space that contains regions with differing dimensionalities.
Also, we generalize from a discrete to a continuous (fractal) dimensionality. This allows for continuous gradients in dimensionality, within the statespace.
In fact, many of the systems we know have a state-space of this poly-dimensional class: these are the complex non-linear dynamic systems like our atmosphere, ecosystems, economies, technological systems, financial systems, etc.
(For semantics sake, one can also state that a state-space does have a single dimensionality and has regions in which the state occupies a lower dimensionality.)
Now we can consider state-spaces with a single dimensionality as a special class of poly-dimensional state-spaces.

Microstate density

A poly-dimensional state-space can show areas where dimensionality (complexity) increases (or decreases) relatively fast. This means that, in that region of the state-space, there are much more micro-states that correspond to a certain macrostate, or rather: a region with a *high density* of available microstates.

The inevitability of entropy production maximization

As stated above, in any state-space the state of the system tends towards a macrostate with more microstates. In a single-dimensional state-space this dynamic is called 'dissipation'. But in a poly-dimensional state-space a region with a higher dimensionality also provides macrostates with more microstates.
These areas are visualized in Figure 2. It clearly shows these 'waves' of micro-state density.
So the same statistical inference applies: if the state of a system enters a region with ascending dimensionality (more microstates), the state progresses on this ascent because it has the highest probability to tend towards the

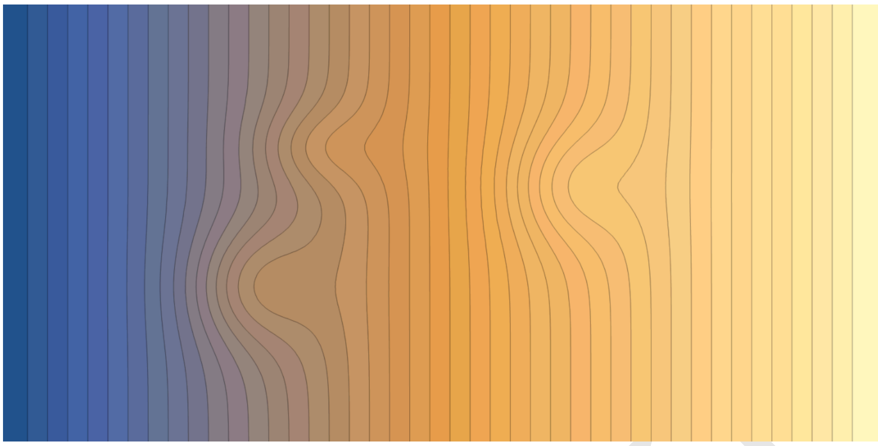

Fig 2: Nonlinear contourmap of W, or micro-state density, based on local complexity-regions (which allow for higher entropy production), and dissipation

state with the most microstates available locally, 'locally' being the key aspect! By definition, this yields the highest amount of entropy production possible.
So it is the same statistical inference as that of entropy maximization, but then in a region of high poly-dimensional density, instead of a region with a single dimension.
This shows the inevitability and ubiquitous validity of entropy production maximization.

Self-organizing criticalities

We have one generalization left, and that is the one towards dynamic state-spaces. All kinds of external and internal dynamics can alter the state-space itself, as the state finds the way of maximum entropy production. Especially the poly-dimensional density-distributions can change: *local maxima* may emerge and disappear. These can be described as bifurcations from unstable to stable attractors (e.g. limit cycles) and vice versa, of non-linear entropy production dynamics. These local maxima, where $\ddot{S} = 0$, are clearly visible in the contourmap in Figure 3 (which is a more extreme version of Figure 2).
It is the set of micro-states $\vec{x}$ which satisfy:

$$\frac{\partial^2 S(\vec{x}, t)}{\partial t^2} = 0 \tag{2}$$

If such a local maximum emerges, and the state of a system is caught in its basin of attraction, we recognize this process as a self-organizing criticality. Eventually such a maximum will disappear again, and this implies a collapse of the Self-Organizing Criticality. For organic systems, this is called death.

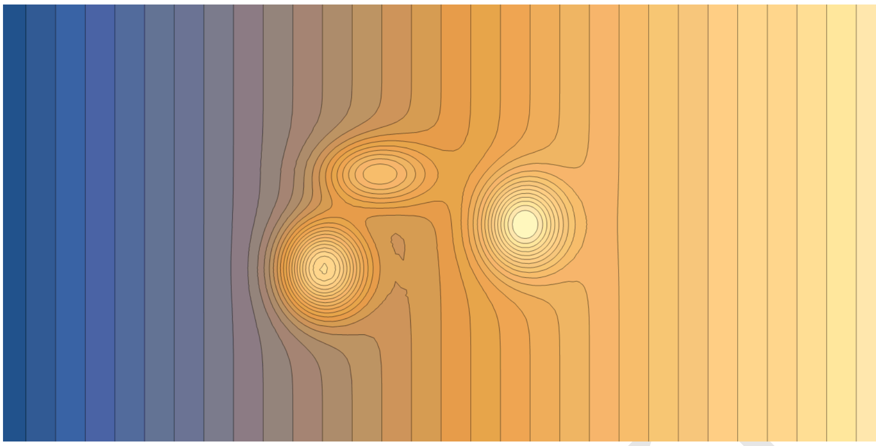

Fig 3: Regions with high gradients of complexity, which allow for self-organizing criticalities

The state decreases in dimensionality again, and resumes its dissipative course, or gets caught in another basis of attraction.
In other words, these areas mark the emergence of what we call 'order', but from this generalization it is actually an *acceleration* towards more disorder. Or rather: order and chaos are formalized into relative concepts (instead of discrete), which should clarify many semantic discussions about what life is.

Dependancy on initial conditions and dynamics

Whether or not the state of a system actually reaches such a self-organizing criticality, depends on its initial condition, as shown in figure 4. But it also depends on the dynamics of the state-space. If you consider the (open) system of a tree, its state-space changes over time, because of changes in the regional ecosystem and in the local weather-conditions, for example. If there is a period of severe drought, existing local maxima can bifurcate away.

Higher-level maximization

The example of the system of a tree is a good example of a fractal superstructure. The dynamics of the state-space of this tree depend on at least its surrounding ecosystem and the weather. But these are also nonlinear dynamic systems, with highly intricate state-spaces with many local maxima of entropy production of their own. And towards the micro-level, all the cells of the tree, and in between its leaves and its seeds, can be seen as subsystems with their own dynamics and state-spaces. So, ultimately, the choice of what you consider a (sub)system is strictly arbitrary, as these dynamics are all related. The same goes for our economic system, or a technological system. All these systems can be seen as superstructures, hierarchical

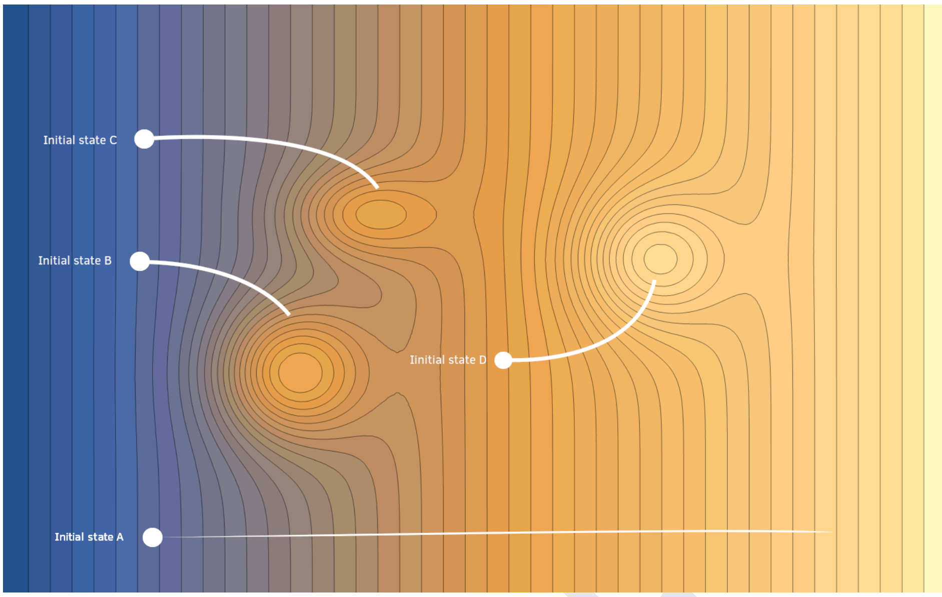

Fig 4: Regions with high gradients of complexity, which allow for self-organizing criticalities

ensembles with some fractal interdependency of non-linear dynamics, from micro to macro-level.
And from the same statistical inference as described above, eventually all these related dynamics will tend towards an optimal distribution of local maxima of entropy production, throughout the total ensemble.

Results and discussion

In this article I have shown that the Maximization of Entropy Production that we observe in many domains, actually follows from the same statistical inference as that of the maximization of entropy itself (i.e. the second law of thermodynamics). It is just a difference between monotonic gradients and local maxima, of microstate-density within the state-space.
This observation has significant impact on our understanding of the world around us, as it applies to many, if not all, domains. Next to the obvious example of meteorology, it explains the 'autopoiesis' of life and other structures. It can even explain many macro-trends in our financial systems, technological domains, our socio-economic domains, and administrative burocracies. Also, many trends in the IT-revolution and globalization should be understood from this perspective: these trends have increased the number of local maxima, and their densities, because of an increase in (local) complexity (dimensionality), which significantly alters existing dynamics with many wanted and unwanted consequences.
Ultimately this can also help in developing much more effective and empiri-

cally based policies at many levels, as every policy is then actually a matter of increasing or decreasing complexity in some subregion, to keep them within some thresholds that are aligned with some normative valuation framework, for example.

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